

A comment on the paper “On the orbit of the LARES satellite”, by I. Ciufolini

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Abstract

In this note we comment on a recent paper by I. Ciufolini about the possibility of placing the proposed terrestrial satellite LARES in a low-altitude, nearly polar orbit in order to measure the general relativistic Lense-Thirring effect with its node. Ciufolini claims that, for a departure of 4 deg in the satellite’s inclination i from the ideal polar configuration ($i = 90$ deg), the impact of the errors in the even zonal harmonics of the geopotential, modelled with EIGEN-GRACE02S, would be nearly zero allowing for a few-percent measurement of the Lense-Thirring effect. Instead, we find that, with the same Earth gravity model and for the same values of the inclination, the upper bound of the systematic error due to the mismodelling in the even zonals amounts to 64% of the relativistic effect investigated.

Keywords: Lense-Thirring effect; Earth gravity field; polar orbits; new satellite

1 The polar configuration for measuring the Lense-Thirring effect

The possibility of measuring the general relativistic gravitomagnetic Lense-Thirring effect by means of the node of a LAGEOS-like satellite placed in a relatively low-altitude ($a \sim 8000$ km), polar ($i \sim 90$ deg) orbit—POLARES in the following—was proposed for the first time by Lucchesi and Paolozzi (2001) and, subsequently, criticized by Iorio (2002). The benefits of such an idea mainly rely in the possibility of using a relatively cheap launcher vehicle and in the fact that, for a perfectly polar configuration ($i = 90$ deg), all the classical secular precessions induced on the node by the even ($\ell = 2, 4, 6, \dots$) zonal ($m = 0$) harmonic coefficients J_ℓ of the Newtonian multipolar expansion of the terrestrial gravitational potential, proportional

to $\cos i$, vanish. The main drawbacks of such an orbital configuration are as follows

- The satellite's node is perturbed, among other things, by the $\ell = 2, m = 1$ constituent of the Solar K_1 tide whose period is equal to that of the spacecraft's node itself: for $i \sim 90$ deg it precesses very slowly, so that K_1 would mimic an aliasing secular trend over an observational time span of a few years compromising the recovery of the genuine relativistic linear trend of interest. This general feature of motion of a polar satellite was already recognized by Peterson (1997) and Iorio (2005a) in the framework of the GP-B mission.
- This problem is avoided by choosing an inclination a few deg apart from the ideal polar configuration. But, in this case, the systematic error $\delta\mu$ induced by the mismodelled part of all the even zonal harmonics, emphasized by the low altitude of the satellite, is enhanced, depending on the accuracy of the gravity model used. Iorio (2002) used the full covariance matrix of EGM96 (Lemoine et al. 1998) up to degree $\ell = 20$ showing that for orbits just 1 deg apart from $i = 90$ deg the impact of the mismodelled even zonals considered amounted to 40%. Such an estimate is likely optimistic because of the use of the correlations among the solved-for even zonals.

In (Iorio 2002) the possibility of using POLARES in conjunction with the existing LAGEOS and LAGEOS II satellites according to the well-known linear combination approach was investigated as well: it turned out to be unfeasible because of the quite large value of the coefficient with which the POLARES node would enter such combinations.

Iorio (2005b) extensively studied the impact of the new Earth gravity models by CHAMP and GRACE on the possibility of using a new satellite to measure the Lense-Thirring effect. Among other things, the POLARES configuration ($a = 8000$ km and $e = 0.04$) was re-analyzed with the EIGEN-CG01C model (Reigber et al., 2006), up to degree $\ell = 20$ and, much more conservatively than in (Iorio 2002), by linearly summing up the absolute values of the individual mismodelled classical precessions; the situation is now improved with respect to the EGM96 case, but it turned out that for a shift of just 2 deg in i with respect to the ideal polar geometry the bias due to the mismodelling in all the uncanceled even zonal harmonics still amounts to about 25%. In Iorio (2005b) also the linear combination scenario with LAGEOS and LAGEOS II was investigated showing that for $a = 8000$ km,

$e = 0.04$, and $60 \text{ deg} < i < 80 \text{ deg}$ the systematic error due to the mismodelled even zonals is 1-3%.

2 The departures from the ideal polar configuration

The subject seemed, thus, to have exhaustively been treated so far, when a new paper on it by Ciufolini (2006) appeared. Basically, the only novelty of such work, which mainly reproduces the content of Section 4.2.1 and Section 4.4 of Iorio (2005b) without quoting it, is a huge underestimation of the impact of the uncertainties in our knowledge of the geopotential on a certain orbital configuration of POLARES. Indeed, Ciufolini (2006), who used the EIGEN-GRACE02S Earth gravity model (Reigber et al., 2005), after discussing the problem of the K_1 tide, proposed to circumvent it by adopting for POLARES an orbital configuration with $a = 7878 \text{ km}$ and $i \leq 86 \text{ deg}$ or $i \geq 94 \text{ deg}$, i.e. an inclination's departure of 4 deg from the ideal polar geometry. He explicitly claims that, in this case “[...] if LARES would be launched in a nearly polar orbit the use of LAGEOS and LAGEOS 2 satellites would not be anymore useful in order to reduce the error budget (and would indeed only introduce an additional error), since the effect of the even zonal harmonics on the node of LARES would be nearly zero, [...]”. Unfortunately, the situation is quite different, as it could already have been inferred from Section 4.2.1 and Figure 5 of (Iorio 2005b). For the sake of a direct comparison, here we use EIGEN-GRACE02S as well, and in order to get a conservative upper bound of the systematic error induced by the mismodelling in J_ℓ we linearly sum up the absolute values of the individual mismodelled node precessions up to $\ell = 40$

$$\delta\mu \leq \sum_{\ell=2}^{40} \left| \dot{\Omega}_{\ell} \right| \delta J_{\ell} \quad (1)$$

by using the calibrated errors in J_ℓ (Reigber et al. 2005). The coefficients $\dot{\Omega}_{\ell}$ of the classical node precessions were explicitly worked out up to degree $\ell = 20$ in (Iorio 2003): for, e.g., $\ell = 2$ we have

$$\dot{\Omega}_{2,2} = -\frac{3}{2}n \left(\frac{R}{a} \right)^2 \frac{\cos i}{(1 - e^2)^2}, \quad (2)$$

where R denotes the Earth's mean equatorial radius and $n = \sqrt{GM/a^3}$ is the Keplerian mean motion. Ciufolini (2006) did not explain how he assessed

the error due to the even zonals (root-sum-square calculation? Sum of the absolute values of the individual errors?), apart from claiming that he used the analytical expressions of the nodal precession of a satellite, up to $\ell = 10$, from an unspecified reference R. Tauraso (2004). For $a = 7878$ km, $e = 0.04$ and $i = 86/94$ deg we get¹

$$\delta\mu \leq 64\%. \quad (3)$$

In Table 1 we release the details of our calculation. As can be noted, the uncanceled precession due to δJ_2 amounts to 70% of the entire error.

In addition to the static part of the geopotential, also its time-dependent components must also be considered. In particular, for $i = 90 \pm 4$ deg, the mismodelled part of the $\ell = 2$ $m = 0$ constituent of the 18.6-year tide would have a serious aliasing impact on a sought few-percent measurement, especially over an observational time span of just 3 years, as proposed by Ciufolini (2006). The uncanceled secular variations of the even zonals $\dot{J}_2, \dot{J}_4, \dot{J}_6$ would be another source of systematic error.

Thus, it seems to us very difficult to agree with the conclusion by Ciufolini (2006) “A nearly polar orbit for LARES at an altitude of about 1500 km would be suitable for a measurement of the Lense-Thirring effect with accuracy of a few percent.”. A new satellite can be fruitfully used only in conjunction with LAGEOS and LAGEOS II. Such existing satellites, however, would set the total realistic accuracy obtainable to a few percent level because of the impact of the non-gravitational forces acting on them, independently of how well they could be reduced on LARES. Indeed, it is not clear if and how LAGEOS and LAGEOS II could benefit from the reduction of the non-gravitational forces on LARES. Such interesting technological and engineering efforts (Bellettini et al. 2006) could likely turn out to be really and fully useful if the launch of at least two entirely new spacecraft was implemented (Iorio 2005b).

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¹If we truncate the calculation at $\ell = 20$ and $\ell = 10$ we get $\delta\mu \leq 63\%$ and $\delta\mu \leq 59\%$, respectively.

Table 1: Individual mismodelled node precessions $\delta\dot{\Omega}^{(J_\ell)} \equiv \left| \dot{\Omega}_{\cdot\ell} \right| \delta J_\ell$ induced by the calibrated errors in J_ℓ , $\ell = 2, 4, 6 \dots 40$, in milliarcseconds per year (mas yr^{-1}), according to the variance matrix of EIGEN-GRACE02S Earth gravity model (Reigber et al. 2005) for $a = 7878$ km, $e = 0.04$, $i = 86$ deg. The mismodelled precessions for $\ell \geq 30$ are smaller than 0.1 mas yr^{-1} . The Lense-Thirring effect for such an orbital configuration amounts to $116.6 \text{ mas yr}^{-1}$. The upper bound of the total error $\delta\mu \leq \sum_{\ell=2}^{40} \delta\dot{\Omega}^{(J_\ell)}$ is quoted, in mas yr^{-1} , in the last row: it amounts to 64% of the Lense-Thirring effect. The most important contribution comes from J_2 whose mismodelled precession amounts to 70% of the total error.

Degree ℓ	$\delta\dot{\Omega}^{(J_\ell)}$ (mas yr^{-1})
2	47.9
4	5.8
6	3.4
8	2.4
10	3.0
12	1.5
14	1.1
16	0.7
18	0.5
20	0.4
22	0.3
24	0.2
26	0.1
28	0.1
30	—
32	—
34	—
36	—
38	—
40	—
$\delta\mu$	68.1

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